

Ada and Alan's Theoretical Machines

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Motivating questions

Ada Lovelace is often called
“the first computer programmer.”

Alan Turing is often called
“the father of theoretical computer science.”

In each case, the argument for the title is largely based on
a single paper.

What is in these papers?

Are the titles reasonable?

Ada Lovelace

Augusta Ada King, Countess of Lovelace (1815-1852)

Daughter of Lord Byron and
Anne Isabella Byron

Received tutoring in mathematics

Met Charles Babbage in 1833

Translated Luigi Menebrea's *Sketch of the Analytical Engine Invented by Charles Babbage* and appended notes. (1842-1843).



Portrait of Ada Lovelace by Margaret Sarah Carpenter

Babbage's Difference Engine

Goal: Generate accurate tables of mathematical values using Newton's finite difference method.

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$$\begin{array}{ccccccc} & & & 2 & 2 & 2 & 2 \\ & & 3 & & & & \\ & 1 & & & & & \end{array}$$

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$$\begin{array}{ccccccc} & & & 2 & & 2 & & 2 & & 2 \\ & & & & & & & & & & \\ & & & 3 & & 5 & & & & & \\ & & & & & & & & & & \\ & & & & & & & & & & \\ & & & & & & & & & & \\ 1 & & & & & & & & & & \end{array}$$

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$$\begin{array}{ccccccc} & & 2 & & 2 & & 2 & & 2 \\ & & & & & & & & & \\ & & 3 & & 5 & & 7 & & 9 & \\ & & & & & & & & & \\ & & & & & & & & & \\ & & & & & & & & & \\ 1 & & & & & & & & & \end{array}$$

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		2	2	2	2
	3		5	7	9
1		4	9	16	

The Difference Engine: Writing your own program

Goal: Write a program to evaluate $p(.1), p(.2), \dots$ where $p(x) = x^2 + x + 1$.

$$p(1) = 1.11 \quad p(2) = 1.24 \quad p(3) = 1.39 \quad p(4) = 1.56$$

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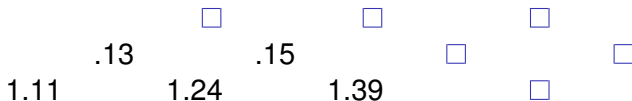
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		.02				
	.13		.15			
1.11		1.24		1.39		

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		.02		.02		.02	
	.13		.15		.17		.19
1.11		<input type="checkbox"/>		<input type="checkbox"/>		<input type="checkbox"/>	

The Difference Engine: Writing your own program

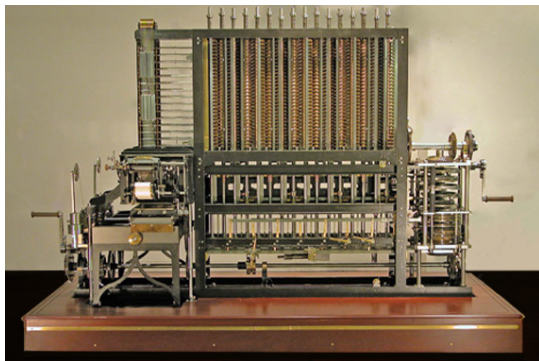
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		.02		.02		.02	
	.13		.15		.17		.19
1.11		1.24		1.39		1.56	

The Difference Engine: The machine

This 11 foot long model of the Babbage's Difference Engine (second design) was completed in London in 2002. An identical engine was completed in 2008 and is on display in the Computer History Museum in Mountain View, California.



Babbage's Analytical Engine

Design features:

Storage cylinders for numbers

Mills for operating on pairs of numbers (+, −, ×, ÷)

Instructions move numbers to a mill and store the output

Sequences of instructions given on cards (Jacquard loom)

Babbage's Analytical Engine: Ada's note D

This program computes a solution to the system of equations:

$$mx + ny = d \quad m'x + n'y = d'$$

Table accompanying Translator's Note D of *Sketch of the Analytical Engine*

Number of Operations Nature of Operations		Variables for Data					Working Variables								Variables for Results		
		¹ V ₀	¹ V ₁	¹ V ₂	¹ V ₃	¹ V ₄	⁰ V ₆	⁰ V ₇	⁰ V ₈	⁰ V ₉	⁰ V ₁₀	⁰ V ₁₁	⁰ V ₁₂	⁰ V ₁₃	⁰ V ₁₄	⁰ V ₁₅	⁰ V ₁₆
		+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+
		0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
		0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
		0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
		0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
		m	n	d	m'	n'	d'										
1	×	m	n'	mn'									
2	×	n	m'	$m'n$									
3	×	d	dn'									
4	×	0	d'	$d'n$									
5	×	0	$d'm$									
6	×	0	0	dm'								
7	−	0	0	$(mn' - m'n)$							
8	−	0	0	$(dn' - d'n)$						
9	−	0	0	$(d'm - dm')$					
10	÷	$(mn' - m'n)$	0	$\frac{dn' - d'n}{mn' - m'n} = x$				
11	÷	0	0	$\frac{d'm - dm'}{mn' - m'n} = y$			

Babbage's Analytical Engine: Ada's note G

This is a program for calculating Bernoulli numbers:

Number of Operation	Nature of Operation	Variables acted upon	Variables receiving results	Indication of change in the value on any Variable	Statement of Results	Data											Working Variables			Result Variables														
						¹ V ₁	¹ V ₂	¹ V ₃	⁰ V ₄	⁰ V ₅	⁰ V ₆	⁰ V ₇	⁰ V ₈	⁰ V ₉	⁰ V ₁₀	⁰ V ₁₁	⁰ V ₁₂	⁰ V ₁₃ ...	¹ V ₂₁	¹ V ₂₂	¹ V ₂₃	⁰ V ₂₄ ...												
						○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○											
						1	2	4	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	
						□	□	□	□	□	□	□	□	□	□	□	□	□	□	□	□	□	□	□	□	□	□	□	□	□	□	□	□	
						B ₁	B ₂	B ₃	B ₄	B ₅	B ₆	B ₇	B ₈	B ₉	B ₁₀	B ₁₁	B ₁₂	B ₁₃	B ₁₄	B ₁₅	B ₁₆	B ₁₇	B ₁₈	B ₁₉	B ₂₀	B ₂₁	B ₂₂	B ₂₃	B ₂₄	B ₂₅	B ₂₆	B ₂₇		
1	x	¹ V ₂ × ¹ V ₃	¹ V ₆ , ¹ V ₉ , ¹ V ₆	$\left\{ \begin{matrix} \textcircled{\text{1}} \textcircled{\text{2}} \textcircled{\text{3}} & = \textcircled{\text{1}} \textcircled{\text{2}} \\ \textcircled{\text{3}} \textcircled{\text{4}} & = \textcircled{\text{1}} \textcircled{\text{2}} \\ \textcircled{\text{4}} \textcircled{\text{5}} & = \textcircled{\text{2}} \textcircled{\text{4}} \end{matrix} \right.$	$= 2n \dots \dots \dots$		2	n	2n	2n	2n																							
2	-	¹ V ₆ - ¹ V ₃	² V ₄	$\left\{ \begin{matrix} \textcircled{\text{3}} \textcircled{\text{4}} & = \textcircled{\text{2}} \textcircled{\text{4}} \\ \textcircled{\text{4}} \textcircled{\text{5}} & = \textcircled{\text{2}} \textcircled{\text{4}} \end{matrix} \right.$	$= 2n - 1 \dots \dots \dots$		1		2n - 1																									
3	+	¹ V ₆ + ¹ V ₃	² V ₅	$\left\{ \begin{matrix} \textcircled{\text{3}} \textcircled{\text{4}} & = \textcircled{\text{2}} \textcircled{\text{4}} \\ \textcircled{\text{4}} \textcircled{\text{5}} & = \textcircled{\text{1}} \textcircled{\text{2}} \end{matrix} \right.$	$= 2n + 1 \dots \dots \dots$		1		2n + 1																									
4	+	² V ₅ + ² V ₄	¹ V ₁₁	$\left\{ \begin{matrix} \textcircled{\text{3}} \textcircled{\text{4}} & = \textcircled{\text{0}} \textcircled{\text{5}} \\ \textcircled{\text{4}} \textcircled{\text{5}} & = \textcircled{\text{0}} \textcircled{\text{5}} \end{matrix} \right.$	$= \frac{2n-1}{2n+1} \dots \dots \dots$				0	0																								
5	+	¹ V ₁₁ + ¹ V ₉	² V ₁₃	$\left\{ \begin{matrix} \textcircled{\text{3}} \textcircled{\text{4}} & = \textcircled{\text{1}} \textcircled{\text{2}} \\ \textcircled{\text{4}} \textcircled{\text{5}} & = \textcircled{\text{1}} \textcircled{\text{2}} \end{matrix} \right.$	$= \frac{1}{2} \cdot \frac{2n-1}{2n+1} \dots \dots \dots$		2																											
6	-	⁰ V ₁₃ - ² V ₁₃	¹ V ₁₃	$\left\{ \begin{matrix} \textcircled{\text{3}} \textcircled{\text{4}} & = \textcircled{\text{0}} \textcircled{\text{5}} \\ \textcircled{\text{4}} \textcircled{\text{5}} & = \textcircled{\text{1}} \textcircled{\text{2}} \end{matrix} \right.$	$= -\frac{1}{2} \cdot \frac{2n-1}{2n+1} = A_0 \dots \dots \dots$																													
7	-	¹ V ₄ - ¹ V ₃	¹ V ₁₀	$\left\{ \begin{matrix} \textcircled{\text{3}} \textcircled{\text{4}} & = \textcircled{\text{1}} \textcircled{\text{2}} \\ \textcircled{\text{4}} \textcircled{\text{5}} & = \textcircled{\text{1}} \textcircled{\text{2}} \end{matrix} \right.$	$= n - 1 (= 3) \dots \dots \dots$		1		n																									
8	+	¹ V ₂ + ⁰ V ₉	¹ V ₇	$\left\{ \begin{matrix} \textcircled{\text{3}} \textcircled{\text{4}} & = \textcircled{\text{1}} \textcircled{\text{2}} \\ \textcircled{\text{4}} \textcircled{\text{5}} & = \textcircled{\text{1}} \textcircled{\text{2}} \end{matrix} \right.$	$= 2 + 0 = 2 \dots \dots \dots$		2																											
9	+	¹ V ₆ + ¹ V ₃	³ V ₁₁	$\left\{ \begin{matrix} \textcircled{\text{3}} \textcircled{\text{4}} & = \textcircled{\text{1}} \textcircled{\text{2}} \\ \textcircled{\text{4}} \textcircled{\text{5}} & = \textcircled{\text{1}} \textcircled{\text{2}} \end{matrix} \right.$	$= 2n = A_1 \dots \dots \dots$				2n	2																								
10	x	³ V ₂₁ × ³ V ₂₁	¹ V ₁₂	$\left\{ \begin{matrix} \textcircled{\text{3}} \textcircled{\text{4}} & = \textcircled{\text{1}} \textcircled{\text{2}} \\ \textcircled{\text{4}} \textcircled{\text{5}} & = \textcircled{\text{1}} \textcircled{\text{2}} \end{matrix} \right.$	$= B_1 \cdot 2n = B_1 A_1 \dots \dots \dots$																													
11	+	¹ V ₁₂ + ¹ V ₁₃	² V ₁₃	$\left\{ \begin{matrix} \textcircled{\text{3}} \textcircled{\text{4}} & = \textcircled{\text{0}} \textcircled{\text{5}} \\ \textcircled{\text{4}} \textcircled{\text{5}} & = \textcircled{\text{1}} \textcircled{\text{2}} \end{matrix} \right.$	$= -\frac{1}{2} \cdot \frac{2n-1}{2n+1} + B_1 \cdot 2n \dots \dots \dots$																													
12	-	¹ V ₁₀ - ¹ V ₃	² V ₁₀	$\left\{ \begin{matrix} \textcircled{\text{3}} \textcircled{\text{4}} & = \textcircled{\text{1}} \textcircled{\text{2}} \\ \textcircled{\text{4}} \textcircled{\text{5}} & = \textcircled{\text{1}} \textcircled{\text{2}} \end{matrix} \right.$	$= n - 2 (= 2) \dots \dots \dots$		1		n - 2																									
13	-	¹ V ₆ - ¹ V ₃	² V ₆	$\left\{ \begin{matrix} \textcircled{\text{3}} \textcircled{\text{4}} & = \textcircled{\text{2}} \textcircled{\text{4}} \\ \textcircled{\text{4}} \textcircled{\text{5}} & = \textcircled{\text{1}} \textcircled{\text{2}} \end{matrix} \right.$	$= 2n - 1 \dots \dots \dots$		1		2n - 1																									
14	-	¹ V ₁ + ¹ V ₇	² V ₇	$\left\{ \begin{matrix} \textcircled{\text{3}} \textcircled{\text{4}} & = \textcircled{\text{1}} \textcircled{\text{2}} \\ \textcircled{\text{4}} \textcircled{\text{5}} & = \textcircled{\text{2}} \textcircled{\text{4}} \end{matrix} \right.$	$= 2 + 1 = 3 \dots \dots \dots$		1																											
15	0	⁰ V ₉ + ⁰ V ₇	¹ V ₈	$\left\{ \begin{matrix} \textcircled{\text{3}} \textcircled{\text{4}} & = \textcircled{\text{2}} \textcircled{\text{4}} \\ \textcircled{\text{4}} \textcircled{\text{5}} & = \textcircled{\text{2}} \textcircled{\text{4}} \end{matrix} \right.$	$= \frac{2n-1}{2n+1} \dots \dots \dots$				2n - 1	3																								
16	x	¹ V ₈ × ⁰ V ₉	⁴ V ₁₃	$\left\{ \begin{matrix} \textcircled{\text{3}} \textcircled{\text{4}} & = \textcircled{\text{0}} \textcircled{\text{5}} \\ \textcircled{\text{4}} \textcircled{\text{5}} & = \textcircled{\text{2}} \textcircled{\text{4}} \end{matrix} \right.$	$= 2n \cdot \frac{2n-1}{2n+1} \dots \dots \dots$																													
17	-	² V ₆ - ¹ V ₃	³ V ₆	$\left\{ \begin{matrix} \textcircled{\text{3}} \textcircled{\text{4}} & = \textcircled{\text{2}} \textcircled{\text{4}} \\ \textcircled{\text{4}} \textcircled{\text{5}} & = \textcircled{\text{1}} \textcircled{\text{2}} \end{matrix} \right.$	$= 2n - 2 \dots \dots \dots$		1		2n - 2																									
18	+	¹ V ₁ + ² V ₇	³ V ₇	$\left\{ \begin{matrix} \textcircled{\text{3}} \textcircled{\text{4}} & = \textcircled{\text{2}} \textcircled{\text{4}} \\ \textcircled{\text{4}} \textcircled{\text{5}} & = \textcircled{\text{1}} \textcircled{\text{2}} \end{matrix} \right.$	$= 3 + 1 = 4 \dots \dots \dots$		1																											
19	+	² V ₆ + ² V ₇	¹ V ₉	$\left\{ \begin{matrix} \textcircled{\text{3}} \textcircled{\text{4}} & = \textcircled{\text{2}} \textcircled{\text{4}} \\ \textcircled{\text{4}} \textcircled{\text{5}} & = \textcircled{\text{2}} \textcircled{\text{4}} \end{matrix} \right.$	$= \frac{2n-2}{2n+1} \dots \dots \dots$				2n - 2	4																								
20	x	¹ V ₉ × ⁴ V ₁₁	⁵ V ₁₁	$\left\{ \begin{matrix} \textcircled{\text{3}} \textcircled{\text{4}} & = \textcircled{\text{0}} \textcircled{\text{5}} \\ \textcircled{\text{4}} \textcircled{\text{5}} & = \textcircled{\text{2}} \textcircled{\text{4}} \end{matrix} \right.$	$= 2n \cdot \frac{2n-1}{2n+1} \cdot \frac{2n-2}{2n+1} = A_2 \dots \dots \dots$																													
21	x	¹ V ₂₂ × ¹ V ₁₁	⁰ V ₁₂	$\left\{ \begin{matrix} \textcircled{\text{3}} \textcircled{\text{4}} & = \textcircled{\text{1}} \textcircled{\text{2}} \\ \textcircled{\text{4}} \textcircled{\text{5}} & = \textcircled{\text{1}} \textcircled{\text{2}} \end{matrix} \right.$	$= B_2 \cdot 2n = B_2 A_2 \dots \dots \dots$																													
22	+	² V ₁₃ + ² V ₁₃	³ V ₁₃	$\left\{ \begin{matrix} \textcircled{\text{3}} \textcircled{\text{4}} & = \textcircled{\text{0}} \textcircled{\text{5}} \\ \textcircled{\text{4}} \textcircled{\text{5}} & = \textcircled{\text{1}} \textcircled{\text{2}} \end{matrix} \right.$	$= A_0 + B_1 A_1 + B_2 A_2 \dots \dots \dots$																													
23	-	¹ V ₁₀ - ¹ V ₃	² V ₁₀	$\left\{ \begin{matrix} \textcircled{\text{3}} \textcircled{\text{4}} & = \textcircled{\text{1}} \textcircled{\text{2}} \\ \textcircled{\text{4}} \textcircled{\text{5}} & = \textcircled{\text{1}} \textcircled{\text{2}} \end{matrix} \right.$	$= n - 3 (= 1) \dots \dots \dots$		1																											
Here follows a repetition of Operations thirteen to twenty-three																																		
24	+	⁴ V ₁₃ + ⁰ V ₂₁	¹ V ₂₄	$\left\{ \begin{matrix} \textcircled{\text{3}} \textcircled{\text{4}} & = \textcircled{\text{0}} \textcircled{\text{5}} \\ \textcircled{\text{4}} \textcircled{\text{5}} & = \textcircled{\text{1}} \textcircled{\text{2}} \end{matrix} \right.$	$= B_7 \dots \dots \dots$																													
25	+	¹ V ₁ + ¹ V ₃	¹ V ₃	$\left\{ \begin{matrix} \textcircled{\text{3}} \textcircled{\text{4}} & = \textcircled{\text{1}} \textcircled{\text{2}} \\ \textcircled{\text{4}} \textcircled{\text{5}} & = \textcircled{\text{1}} \textcircled{\text{2}} \\ \textcircled{\text{6}} \textcircled{\text{7}} & = \textcircled{\text{0}} \textcircled{\text{5}} \\ \textcircled{\text{8}} \textcircled{\text{9}} & = \textcircled{\text{2}} \textcircled{\text{4}} \end{matrix} \right.$	$= n + 1 = 4 + 1 = 5$ by a Variable-card. by a Variable-card.		1	n + 1		0	0																							



Summary: Ada Lovelace's theoretical computing

The Analytical Engine was never built.

Program control was limited. (Loops.)

Lovelace and Babbage collaborated on creating instruction sets for the Analytical Engine.

Was Ada Lovelace the first computer programmer?

Alan Turing

Alan M. Turing (1912-1954)

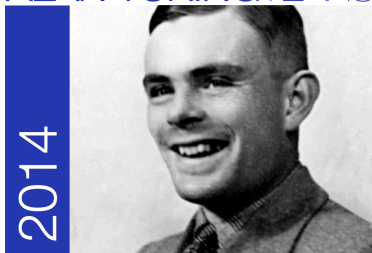
Cambridge undergraduate
and fellow

PhD from Princeton

Thesis advisor: Alonzo Church

Published *On computable numbers with an application to the entscheidungsproblem* in the Proceedings of the London Mathematical Society

ALAN TURING YEARS



Turing machines

Excerpt from *On computable numbers* . . .

<i>Configuration</i>		<i>Behaviour</i>	
<i>m-config.</i>	<i>symbol</i>	<i>operations</i>	<i>final m-config.</i>
b	None	$P0, R$	c
c	None	R	c
e	None	$P1, R$	f
f	None	R	b

b

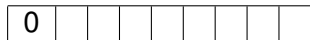


Turing machines

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c	None	R	c
e	None	$P1, R$	f
f	None	R	b

C

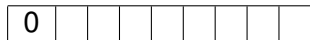


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<i>Configuration</i>		<i>Behaviour</i>	
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b	None	$P0, R$	c
c	None	R	c
e	None	$P1, R$	f
f	None	R	b

e



Turing machines

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<i>Configuration</i>		<i>Behaviour</i>	
<i>m-config.</i>	<i>symbol</i>	<i>operations</i>	<i>final m-config.</i>
b	None	$P0, R$	c
c	None	R	c
e	None	$P1, R$	f
f	None	R	b

f

0		1							
---	--	---	--	--	--	--	--	--	--

Turing machines

Excerpt from *On computable numbers* . . .

<i>Configuration</i>		<i>Behaviour</i>	
<i>m-config.</i>	<i>symbol</i>	<i>operations</i>	<i>final m-config.</i>
b	None	$P0, R$	c
c	None	R	c
e	None	$P1, R$	f
f	None	R	b

b



Turing machines

Excerpt from *On computable numbers* . . .

<i>Configuration</i>		<i>Behaviour</i>	
<i>m-config.</i>	<i>symbol</i>	<i>operations</i>	<i>final m-config.</i>
b	None	$P0, R$	c
c	None	R	c
e	None	$P1, R$	f
f	None	R	b

C

0		1		0				
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Another Turing machine

Another excerpt from *On computable numbers* . . .

<i>Configuration</i>		<i>Behaviour</i>	
<i>m-config.</i>	<i>symbol</i>	<i>operations</i>	<i>final m-config.</i>
b		$P\emptyset, R, P\emptyset, R, P0, R, R, P0, L, L$	c
c	{ 1	R, Px, L, L, L	c
	{ 0		q
q	{ Any (0 or 1)	R, R	q
	{ None	$P1, L$	p
p	{ x	E, R	q
	{ \emptyset	R	f
	{ None	L, L	p
f	{ Any	R, R	f
	{ None	$P0, L, L$	c

This machine computes 0010110111011110 . . .

More ideas from Turing's paper

- Every Turing machine has a “description number” that could be written on a tape.
- There is a *universal machine* that given any tape containing the description number for a machine P prints the output of P .

More ideas from Turing's paper

- Every Turing machine has a “description number” that could be written on a tape.
- There is a *universal machine* that given any tape containing the description number for a machine P prints the output of P .
- There is no machine that given any tape containing the description number for a machine P either prints 1 and halts if P halts or prints 0 and halts if P does not halt.

The halting problem is not Turing computable.

Hilbert's *entscheidungsproblem*

Turing also solved Hilbert's "decision problem" by proving:

There is no machine that given any tape containing the description number for a formula of predicate calculus either prints 1 and halts if the formula is logically valid or prints 0 and halts if the formula is not logically valid.

This claim implicitly relies on the equivalence of Hilbert's notion of "algorithmically computable" and "computable by a Turing machine."

Summary: Alan Turing's theoretical computing

- Turing created a formalization of computing that is still commonly used.
- Formalizations created by other logicians (e.g. Church and Kleene) are equivalent.
- Turing's universal machine foreshadowed the creation of *general purpose computers* like ENIAC (1946).

Was Alan Turing the father of theoretical computer science?

Some references

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- [4] Alan M. Turing, *On computable numbers with an application to the entscheidungsproblem*, Proc. Lond. Math. Soc. **42** (1937), 230–265.