Reverse Mathematics of Free Sets

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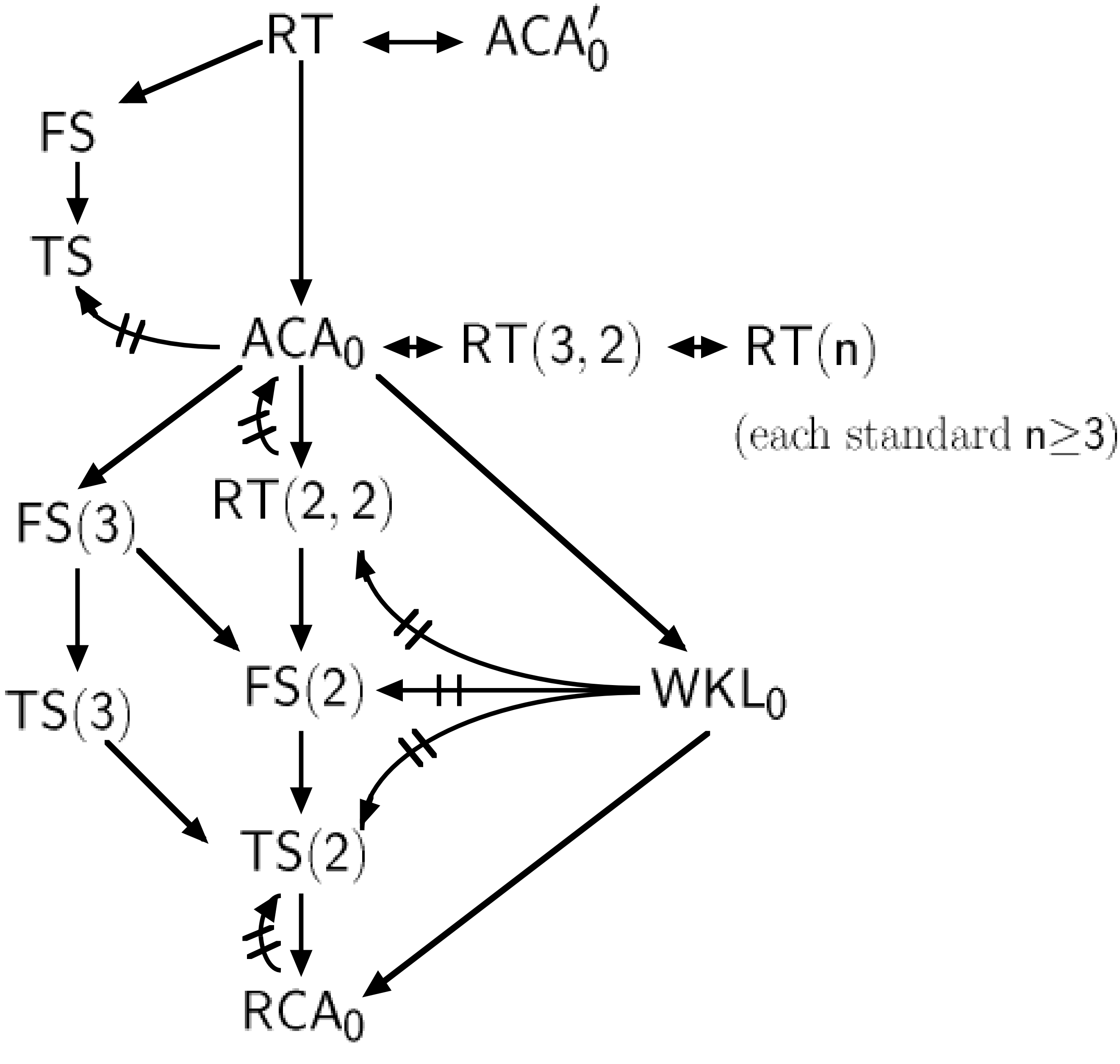
These slides appear at
`www.mathsci.appstate.edu/~jlh`



RT
ACA_0'
FS
TS
ACA_0
RT(3, 2)
RT(n)
(each standard n≥3)
FS(3)
RT(2, 2)
TS(3)
FS(2)
WKL_0
TS(2)
RCA_0

$\mathsf{RCA_0}$ Second order arithmetic with comprehension for computable sets

$\mathsf{ACA_0}$ $\mathsf{RCA_0}$ plus arithmetical comprehension

$\mathsf{ACA_0'}$ $\mathsf{ACA_0}$ plus $0^{(n)}$ exists for every n

$\mathsf{WKL_0}$ $\mathsf{RCA_0}$ plus König's lemma for 0-1 trees

RT Ramsey's theorem: If $f : [\mathbb{N}]^n \to k$ then there is an infinite X such that f is constant on $[X]^n$.

$\mathsf{RT}(3,2)$ Ramsey's theorem restricted to $n = 3$ and $k = 2$

FS Free Set Theorem: If $f : [\mathbb{N}]^n \to \mathbb{N}$ then there is an infinite X such that $\vec{x} \in X$ and $f(\vec{x}) \in X$ imply $f(\vec{x}) \in \vec{x}$.

$\mathsf{FS}(3)$ Free set theorem for $n = 3$

TS Thin Set Theorem: If $f : [\mathbb{N}]^n \to \mathbb{N}$ then there is an infinite X such that $f([X]^n) \neq \mathbb{N}$.

References

Proof that WKL_0 doesn't prove $\mathsf{FS}(2)$:
Jockusch (private communication) based on Theorem 3.1 of his article *Ramsey's theorem and recursion theory,* JSL (37) 1972 page 270.

Proof that ACA_0 doesn't prove TS:
Friedman in *Issues and problems in reverse mathematics,* (Simpson co-author) pages 139-140 of Computability Theory and Its Applications (Cholak, Lempp, Lerman, and Shore editors) AMS 2000.

Other results appear in Cholak, Guisto, and Hirst's *Free sets and reverse mathematics,* to appear in Reverse Mathematics 2001 (Simpson editor). Preprints available at:

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