Fostering International Student Research in the Mathematical Sciences

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Appalachian State University

Research related student international travel

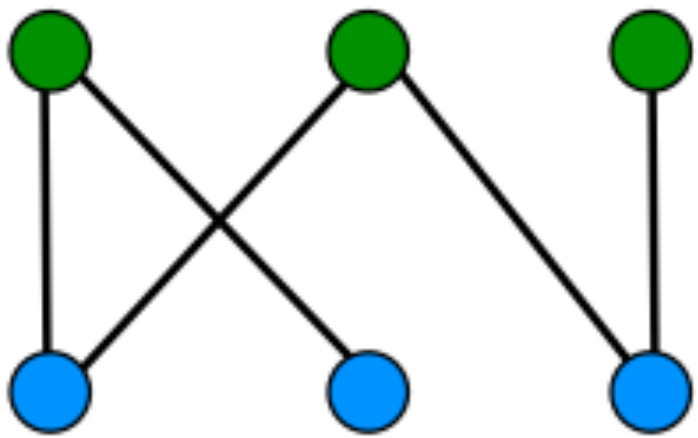
During his senior year, alumnus Noah Hughes gave a talk on his senior honors thesis in the logic seminar at the University of Ghent. Paul Shafer was our contact in Ghent.

An example of student research in mathematics

A formalization of a theorem of Marshall Hall, Jr.:

Theorem

(RCA_0) *If* $M = (B, G)$ *is a finite bipartite graph with unique matching, then there is an enumeration of* B *such that for every* i, $|G(\{b_0, \ldots, b_{i-1}\})| = i$.



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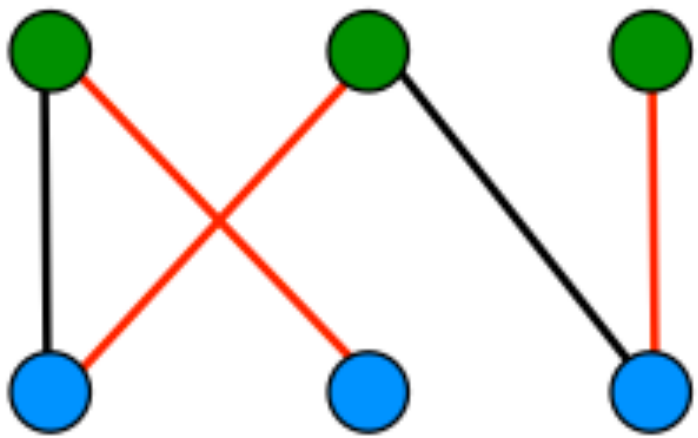

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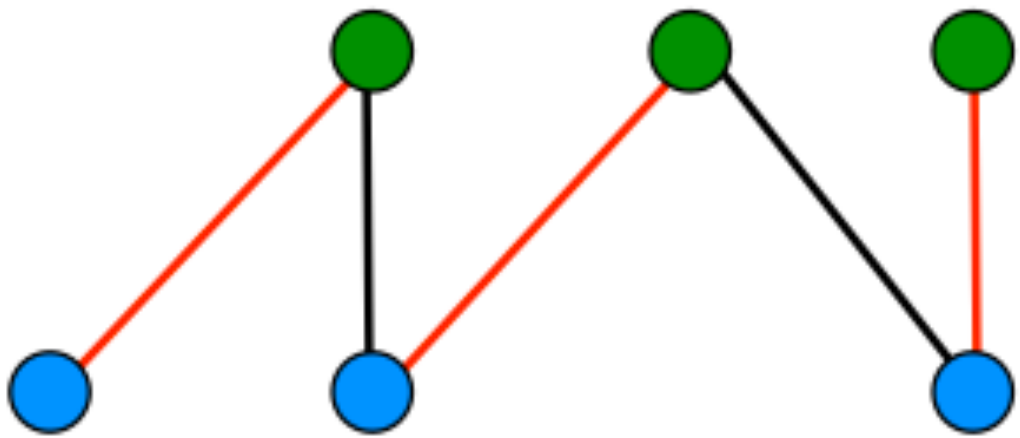


A result about infinite matchings (with Noah)

Theorem

(RCA_0) *The following are equivalent:*

1. WKL_0.
2. *Suppose $M = (B, G)$ is a bipartite graph and $h(b) = |G(b)|$ for every $b \in B$. If M has a unique matching, then there is an enumeration of B such that for every i, $|G(\{b_0, \ldots, b_{i-1}\})| = i$.*

Note: The existence of the enumeration is actually a necessary and sufficient condition for the existence of a unique matching.

Student research related to Ramsey's theorem

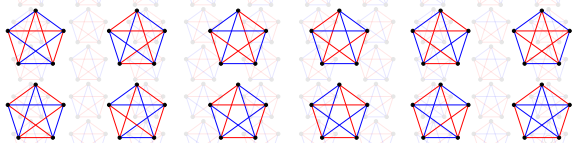
How many 2-colorings of K5 have no 1-colored K3 ?

Ramsey Interest Group: Anthony Hengst, Sergei Miles, Isaac Medina Silva, Allison Staley **Faculty Mentor: Jeff Hirst**

Appalachian State University, Department of Mathematical Sciences, Boone, North Carolina 28608

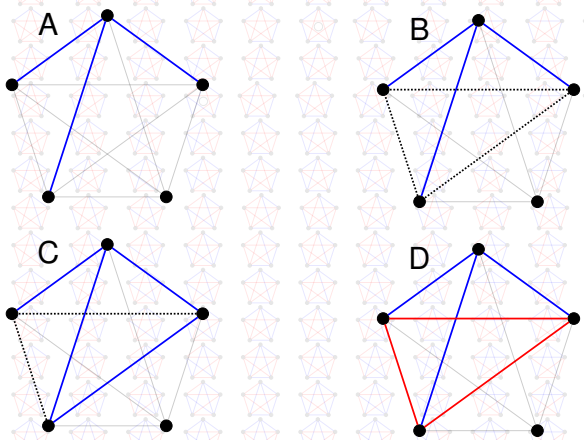
Introduction

Of the 1024 possible 2-colorings of K5, only 12 have no 1-colored triangles.



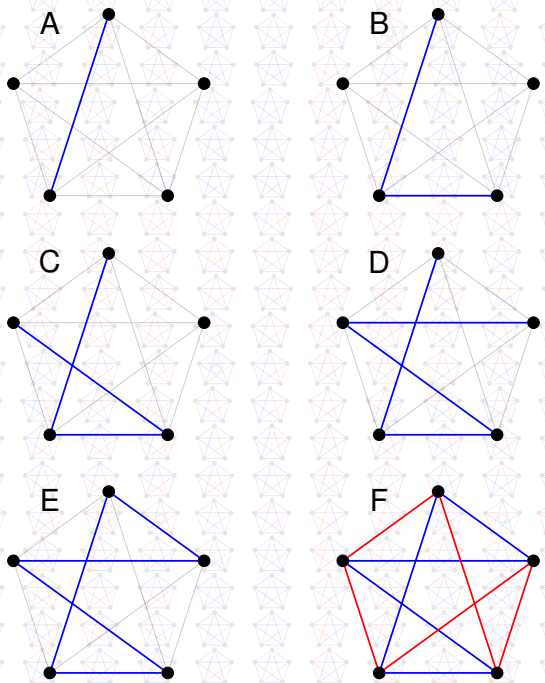
Claim 1

If any 3 edges match, then there is a 1-colored triangle.



Claim 2

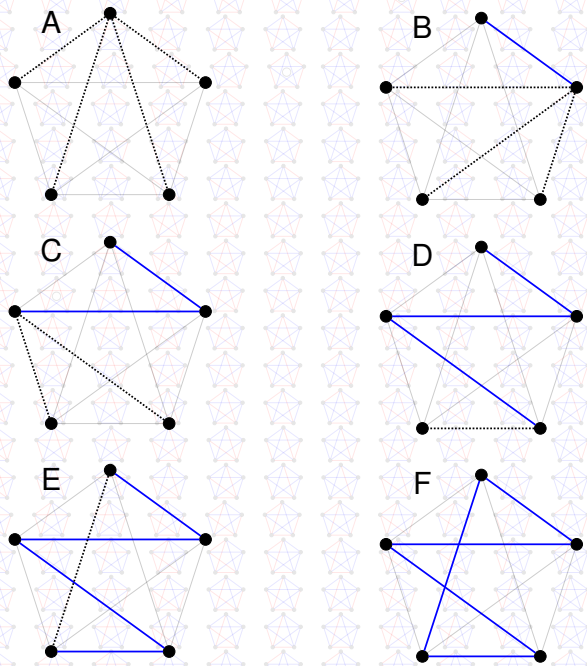
If G has no 1-colored triangles, then G has a 1-colored 5-cycle.



E: 1-colored 5-cycle
F: Remaining edges form a 5-cycle

Claim 3

There are 12 ways to construct a 1-colored 5-cycle.



$$\frac{4 \cdot 3 \cdot 2 \cdot 1 \cdot 1}{2} = 12$$

Making contacts, groundwork

International workshops provide opportunities to create new contacts.

- Smaller than conferences
  - greater interaction
  - disciplinary focus
- More international participants
- Travel tips

Organization of the workshop may or may not be international.

Workshop example 1: Rome

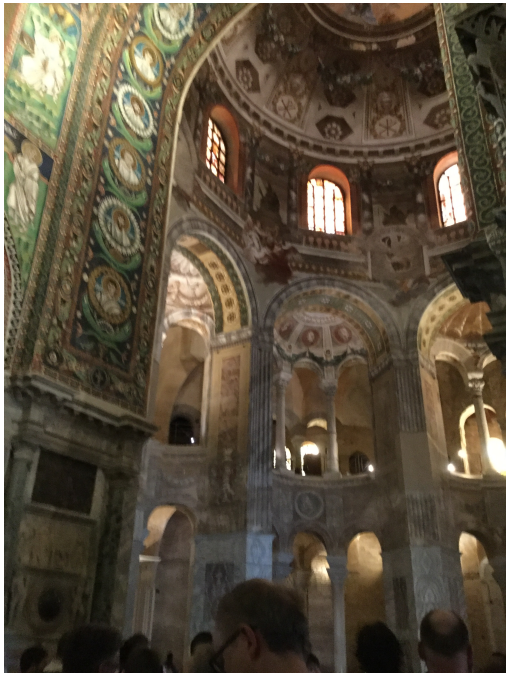
Workshop on Ramsey Theory and Computability
Rome Global Gateway of Notre Dame University
July 9-13, 2018

Participants from:

- Leeds University
- University of Bern
- Central South University of China
- Dartmouth College
- Japan Advanced Institute of Science and Technology
- Università di Roma Sapienza
- Appalachian State
- Cornell University
- Università di Pisa
- National University of Singapore
- University of Vienna
- Swansea University
- University of Pennsylvania
- Università degli Studi di Udine

Workshop example 2: Bertinoro, Italy

RaTLoCC18:
Ramsey Theory in Logic, Combinatorics, and Complexity
Bertinoro International Center for Informatics
July 15-20, 2018
37 participants from Spain, Germany, USA, England, Greece, Czech Republic, Russia, Poland, Italy, Austria, France, and Canada



Basilica of San Vitale in Ravenna

Workshop example 3: Wadern, Germany

Dagstuhl Seminar 18361:
Measuring the Complexity of Computational Content:
From Combinatorial Problems to Analysis
Leibniz-Zentrum für Informatik
September 2-7, 2018
43 participants from Spain, France, USA, Germany, Austria, England, Japan, New Zealand, Italy, Singapore, Chile, and Russia

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- Office of International Education and Development
- Board of Trustees International Research Grants
- Student And Faculty Excellence (SAFE) Fund, College of Arts and Sciences

References:

[1] Jeffry L. Hirst and Noah A. Hughes, *Reverse mathematics and marriage problems with finitely many solutions*, Arch. Math. Logic **55** (2016), no. 7-8, 1015–1024, DOI 10.1007/s00153-016-0509-4. MR3555339

[2] _____, *Reverse mathematics and marriage problems with unique solutions*, Arch. Math. Logic **54** (2015), no. 1-2, 49–57, DOI 10.1007/s00153-014-0401-z. MR3304736